PROBLEMS IN METEOROLOGY.

By C. F. VON HERRMANN, Section Director. Dated Baltimore, Md., June 9, 1906.

[Continued from December, 1906, p. 579.]

Problem 18.—Given a cubic meter of dry air at the temperature of 30° C. under standard pressure, 760 millimeters, and let it become saturated with vapor; find the formula expressing its increase in volume when the pressure and temperature remain constant, and the air continues to be saturated at the final volume.

Solution.—In general, if water evaporates into dry air under constant pressure, the elastic force of the mixture is increased by the amount of the vapor tension, and if allowed to expand the mixture will increase in volume; if sufficient vapor be added to saturate the increased space, then the increased volume (final volume of the saturated air at the given temperature) may be calculated as follows:

Let the mixture expand to the volume V^1, so as to maintain the same pressure, p, and temperature. If the mixture is then saturated with vapor, the new volume will consist of V volumes of dry air, and $V^1 - V$ volumes of aqueous vapor.

Since, by the law of Boyle-Gay Lussac, $p\,V = p^1\,V^1$ (the volume varies inversely as the pressure, or pressure inversely as volume) then the pressure of the volume V of dry air will be $p^1 = p \times V/V^1$. The pressure of $V^1 - V$ volumes of vapor will be e, ($p^1 + e$ must equal p or 760 mm.).

Then the mixture will be composed of

V volumes of air under pressure $p \times V/V^1$

$V^1 - V$ volumes of vapor under pressure e.

The total pressure of the mixture

$$p = e + p \times V/V^1$$
$$V^1 p = V^1 e + p\,V$$
$$V^1 p - V^1 e = p\,V$$
$$V^1 = p\,V/(p - e).$$

Therefore the increased volume is $p/(p - e)$ times the initial volume. If the initial volume is 1 cubic meter, the increased volume is

$$\frac{p}{(p-e)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

(See Abbe's Treatise on Meteorological Apparatus, p. 348.)

One cubic meter of dry air at $t = 30°$ C., pressure = 760 mm., when saturated with aqueous vapor and allowed to expand, will have a final saturated volume of $760/(760 - 31.51)$ which is equal to 1.0432 cubic meters.

The student should be required to construct a table showing what a unit volume of dry air becomes when saturated, at various temperatures.

The weight of vapor in a saturated cubic meter of space at 10° is 9.34 grams. Prove this in another way:

One cubic meter of saturated air at 10° weighs 1241.6 grams.

One cubic meter of dry air at 10° weighs 1247.3 grams. This cubic meter of air, however, in becoming saturated expands to 1.0122 cubic meters (found as above). Therefore, 1 cubic meter of this expanded air weighs only 1232.26 grams.

Therefore, of the 1 cubic meter of saturated air which weighs 1241.6 grams, 1232.26 grams are dry air, and the remaining 9.34 grams are vapor; which agrees with the above.

Problem 19.—What volume of saturated air has the same weight as 1 cubic meter of dry air, both being at the same temperature and pressure?

Solution.—One cubic meter of dry air weighs $\dfrac{1293.05\,b}{(1 + \alpha t)\,760}$ grams $= (A)$.

One cubic meter of saturated air weighs $\dfrac{1293.05\,(b - .378\,e)}{(1 + \alpha t)\,760}$ grams $= (B)$.

Therefore, 1 gram of saturated air occupies the reciprocal of B or

$$\frac{(1 + \alpha t)\,760}{1293.05\,(b - .378\,e)} = n \text{ cubic meters.}$$

If one gram of saturated air occupies n cubic meters, then A grams will occupy $A\,n$ cubic meters or

$$\frac{(1 + \alpha t)\,760 \times 1293.05\,b}{1293.05\,(b - .378\,e)\,(1 + \alpha t)\,760} = \frac{b}{(b - 0.378\,e)}.$$

If the pressure is 760 millimeters and the temperature 0°C., then $760/758.25$ or 1.0023 cubic meters of saturated air will weigh the same as 1 cubic meter of dry air at 0°.

Proof.—One cubic meter of saturated air weighs 1290.08 grams; then 1.0023 cubic meters will weigh 1290.08×1.0023 grams which is exactly equal to 1293.05 grams, the weight of a cubic meter of dry air at 0°C.

Problem 20.—At what temperature will a cubic meter of saturated air have the same weight as a cubic meter of dry air at the temperature t?

Solution.—Let t be the temperature of the dry air and t' that of the saturated air; then the weight of a cubic meter of dry air at t is to be equal to the weight of a cubic meter of saturated air at t' or

$$\frac{1293.05\,(b - .378\,e)}{(1 + \alpha t')\,760} = \frac{1293.05\,b}{(1 + \alpha t)\,760}.$$

Solving for t' gives $t' = t - .378\,(e/b)\,t - 103\,(e/b)$. If we take the simple case where the temperature of the dry air is 0° this gives the temperature of the saturated air of the same weight as $t' = -103\,(e/b)$ which is $-0.62°$ C.

Problem 21.—A cubic meter of dry air at any pressure is mixt with a cubic meter of vapor, the mixture being allowed to expand under a given atmospheric pressure. No more vapor is added, so that at the expanded volume the space is not saturated. What is the increase in the volume of the mixture?

Solution.—We have to calculate the volume of a mixture of 1 cubic meter of dry air under any pressure B with 1 cubic meter of vapor, not necessarily saturated, under the tension e, when the mixture is allowed to expand freely under the pressure b.

The first effect of the mixture may be considered to be the formation in a rigid enclosure of 2 cubic meters of a mixture whose total elastic pressure is $\frac{1}{2}\,(B + e)$. If these two cubic meters are confined, not by a rigid inclosure, but by the atmospheric pressure b, then they assume a new volume V' such that

$$2\left(\frac{B}{2} + \frac{e}{2}\right) = V'\,b \quad \ldots\ldots\ldots\ldots\ldots (1)$$

Hence $$V' = (B + e)/b \quad \ldots\ldots\ldots\ldots\ldots (2)$$

and the increase in volume is

$$V' - 1 = \frac{B + e - b}{b} = \frac{B - b}{b} + \frac{e}{b} \quad \ldots\ldots\ldots (3)$$

Special case.—If B is the same as b, or the pressure of the outside atmosphere, then the solution becomes

$$V' = (b + e)/b = 1 + e/b, \text{ or } V' - 1 = e/b \quad \ldots\ldots (4)$$

in which e has its maximum value when the original cubic meter of vapor is saturated, but the formula and its method of demonstration hold good for any relative humidity.

Illustration.—When 1 cubic meter of saturated vapor at 10° C., for which $e = 9.14$ millimeters, is mixt with 1 cubic meter of dry air at the same temperature and atmospheric pressure of 760 millimeters, the volume of the mixture becomes

$$1 + 9.14/760 \text{ or } 1.0120 \text{ cubic meters.}$$

When 1 cubic meter of dry air at 760 millimeters becomes completely saturated at 10° C., it expands to 1.0122 cubic meters. (See problem 18.) The difference between these two cases is seen to be very small.

Special case.—The student should solve the following: What quantity of dry air in cubic meters under 760 millimeters pressure, when mixt with the quantity of vapor necessary to saturate 1 cubic meter of space, will expand so as to occupy exactly 1 cubic meter and be saturated? Ans. $1 - e/b$ cubic meter.

Problem 22.—What is the relative humidity of the mixture considered in problem 21, and what is the vapor pressure in the special case where a cubic meter of saturated vapor is taken?

Solution.—The mixture contains only the weight of vapor necessary to saturate 1 cubic meter of space, which, by equation (2) problem 13, is

$$\frac{1293.05 \times 0.622\, e}{(1 + a\,t)\ 760} \text{ grams of vapor} \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

But by problem 21, the cubic meters of air and vapor have expanded to $(B+e)/b$ cubic meters, and this increased volume, if saturated, will contain

$$\frac{1293.05 \times 0.622\, e \times (B+e)}{(1+a\,t)\ 760\ b} \text{ grams of vapor} \ldots\ldots\ldots (2)$$

Since relative humidity, exprest as a percentage, is 100 multiplied by the ratio of the quantity of vapor actually present, as given by (1), to the quantity of vapor the expanded volume could contain if saturated, given by (2), we have

$$\text{Relative humidity} = 100\, b/(B+e) \text{ per cent.}$$

Special case.—In a mixture under 760 millimeters pressure, of 1 cubic meter of dry air at 760 millimeters, and 1 cubic meter of saturated vapor, both at 10° C., since $e = 9.14$ millimeters, we have

$$\text{Relative humidity} = 100 \times \frac{760}{760 + 9.14} = 98.8 \text{ per cent.}$$

The actual vapor pressure divided by the saturation vapor pressure is the relative humidity; if this ratio is multiplied by 100, we get the relative humidity exprest as a percentage. If the relative humidity of any mixture is 50 per cent, the vapor pressure is 50/100 of saturation; it is therefore $e \times 50/100$. If the relative humidity, as above, is $100\,b/(B+e)$ per cent, the vapor pressure becomes

$$\frac{e}{100} \times \frac{100\,b}{B+e} = \frac{e\,b}{B+e}$$

In the case given, where $e = 9.14$ mm., the vapor pressure is

$$\frac{9.14 \times 760}{760 + 9.14} = 9.03 \text{ mm.}$$

Problem 23.—What volume will a cubic centimeter of water at temperature 4° C. (when it has its maximum density) occupy in the state of vapor at the temperature of 100° C.?

Solution.—This is a very simple application of the formula of problem 13. A cubic meter of saturated vapor at 100° C. weighs

$$\frac{0.622 \times 1293.05}{1 + (0.00367 \times 100)} \times \frac{760}{760} \text{grams}$$

since at 100° C. the vapor pressure is 1 atmosphere, or 760 millimeters.

This reduces to 588 grams. Then if 1 cubic meter weighs 588 grams, 1 gram at 100° will occupy 1/588 cubic meter or 0.001698 cubic meter, which is 1698 cubic centimeters. One gram of water at its maximum density at 4° C. occupies 1 cubic centimeter; so 1 cubic centimeter of water at 4° C. occupies 1698 cubic centimeters when it becomes vapor at 100°. See Deschanel, page 363, part 284.

Problem 24.—To find the vapor tension, the temperature of the dew-point, the weight of the vapor and the dry air, respectively, and the relative humidity when we know the temperatures of the dry-bulb and wet-bulb thermometers.

This is best done by the use of any psychrometric table, e. g., Marvin's Tables, published by the Weather Bureau. Many examples should be worked out by these tables in order to attain proficiency and secure familiarity. The physical principles on which these tables are based are explained at pages 380–391 of Professor Ferrel's Recent Advances in Meteorology, published as Appendix 71 to the Annual Report of the Chief Signal Officer, 1885. They are also given in Section D of the Treatise on Meteorological Apparatus and Methods, published as Appendix 46 to the Annual Report of the Chief Signal Officer, 1887.

Problem 25.—How much will the air be warmed by the latent heat liberated in the formation of a certain amount of frost?

Solution.—On February 26, 1905, Mr. Seeley melted the frost and found its equivalent to be 0.018 inch in depth of water; 0.018 inch is 0.457 millimeter. Since 1 kilogram (1000 grams) on a square meter of surface is equal to 1 millimeter of water at its maximum density, i. e., at 4° C., then to obtain 0.457 millimeters of precipitation would require 457 grams on a square meter, or the 0.018 inch is equal to 457 grams per square meter.

The amount of heat liberated by the condensation of 1 gram of vapor to water at 0° C. is in round numbers 600 calories, and the freezing of 1 gram of water liberates 80 calories additional, or a total of 680 calories. Hence, 457 grams will liberate 680 × 457 or 310 760 calories.

A cubic meter of air at 0° C. and 760 millimeters weighs 1293.05 grams. The specific heat of air is 0.238; to warm 1293.05 grams of air 1° C. will require 1293 × 0.238 calories or 307.734 calories. As the precipitation liberated 310 760 calories, this is sufficient to warm up 310 760 ÷ 307.734 or 1009.8 cubic meters of air 1° C. This was solved in English measures in the MONTHLY WEATHER REVIEW, April, 1905, page 155, but the following is more accurate:

A column 1009.8 meters high above a square meter of ground becomes in English measures a column 3313.6 feet high above a square foot of ground. Hence, the heat liberated in this column will warm up its 3313.6 cubic feet of air by 1° C. or 1.8° F.; or 331 cubic feet by 18° F., or 596 cubic feet by 10° F.

[The 525 cubic feet given in the April, 1905, REVIEW undoubtedly arose from using 535.9 (more properly 536.6) as the latent heat of vaporization at 0° C. This is really the latent heat of vaporization at the boiling point, and the proper figure for the freezing point is 596.7, to which must be added 80 as the latent heat for melting ice at 0° C., making a total of 676.7. See Watson, Text Book of Physics, fourth edition, page 249. The Editor may be responsible for the error.—EDITOR.]

PROBLEMS IN MIXTURES OF AIR AND VAPOR.

[The general problems of mixtures of vapor and air are solved algebraically by the EDITOR in the following lines, to which Mr. von Herrmann has added notes and numerical examples.]

1. Let n volumes of dry air at pressure p_a, relative humidity 0, and temperature t, mix with n' volumes of aqueous vapor at pressure p_v, relative humidity r, and temperature t, forming within a rigid inclosure $n + n'$ volumes of mixture at pressure p_a' for dry air, and p_v' for vapor, and total pressure p', relative humidity r', and temperature t.

Let e be the saturation tension of the vapor for temperature t. For accurate work both p and e must be exprest in standard units, e. g., the height of a column of mercury under standard temperature and gravity. The law of Boyle states that as long as the temperature remains the same, the product